Hierarchical MUSIC Technique for Direction Finding of Multiple Signal Sources

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***Abstract*—We propose a novel *hierarchical* multiple signal classification (H-MUSIC) technique that elaborately estimates the direction-of-arrival (DOA) of multiple signal sources arriving from similar directions. In general, the conventional MUSIC technique does not result in a good DOA estimation performance in this case. The basic idea of the proposed H-MUSIC technique is to combine the conventional MUSIC and beam-space MUSIC techniques. The receiver performs a coarse DOA estimation using the conventional MUSIC technique as a first step, considering a whole range of directions. Then, in the subsequent step, the receiver additionally performs a fine DOA estimation using the beam-space MUSIC technique, focusing on the directions where the signals are likely to exist. Through extensive computer simulations, we show that the proposed H-MUSIC technique significantly outperforms the conventional MUSIC technique even in the low signal-to-noise (SNR) regions and with a small number of time samples.**

***Index Terms*—Wireless positioning, direction-of-arrival (DOA), direction finding, multiple signal classification (MUSIC).**

I. Introduction

As wireless sensing and positioning are expected to become promising applications in the next-generation communication systems, precise direction finding technologies are necessary not only for a large number of terrestrial terminals but also for non-terrestrial networks with many commercial satellites or unmanned aerial vehicles [1]. In general, direction-of-arrival (DOA) estimation techniques with array antennas have robust performance only when the correlation between signal components is small [2]. For example, multiple signal classification (MUSIC) technique, one of the representative array antenna-based DOA estimation algorithms, is known to be quite sensitive to correlation between incident signals [3].

In this paper, hence, we propose a novel hierarchical MUSIC (H-MUSIC) technique with a stepwise DOA estimation procedure to classify multiple signals. Specifically, the H-MUSIC technique consists of two steps: 1) coarse DOA estimation using the conventional MUSIC technique while considering whole range of directions and 2) fine DOA estimation using beam-space MUSIC [4] for certain ranges where signal sources are observed during the first step. Through computer simulations, we show that the H-MUSIC technique significantly outperforms the conventional MUSIC technique even in the low signal-to-noise (SNR) regions and with small number of time samples.

II. System Model

We consider an environment in which an origin-centered uniform circular array (UCA) receiver composed of M antenna elements estimates the DOAs of K signal sources as shown in Fig. 1. Here, it was assumed that the number of incident signal sources is smaller than the number of antennas, i.e., $M > K$. Then, the received signal matrix, $\mathbf{Y} \left(\in \mathbb{C}^{M\times T}\right)$ for T samples is given by

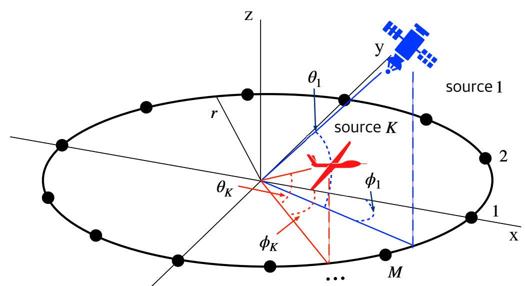

Fig. 1. System model of hierarchical MUSIC where a UCA receiver with M elements and K signal sources exist.

$$\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{N}, \tag{1}$$

where $\mathbf{Y}$ consists of T columns in which $t\,(\in \{1, \cdots, T\})$-th column denotes the received signal vector $\mathbf{y}_t \left(\in \mathbb{C}^{M}\right)$ at t-th time, $\mathbf{A}\left(\in \mathbb{C}^{M\times K}\right)$ denotes the matrix concatenating the steering vectors of each incident signal source, i.e., $\mathbf{A} = [\mathbf{a}\left(\phi_1, \theta_1\right)\ \mathbf{a}\left(\phi_2, \theta_2\right)\ \cdots\ \mathbf{a}\left(\phi_K, \theta_K\right)]$. Here, it is also assumed that the direction of signal sources does not change over T sample period. Thus, $\mathbf{a}(\phi_k, \theta_k)\left(\in \mathbb{C}^{M}\right)$ only indicates the steering vector of $k \in \{1, \cdots, K\}$-th source parameterize the azimuth angle $\phi_k(\in\ (0, 2\pi])$ and the elevation angle $\theta_k(\in\ [0, \pi/2])$. Then, the $m \in \{1, \cdots, M\}$-th element of steering vector, $a_m(\phi_k, \theta_k)$, is defined as

$$a_m(\phi_k, \theta_k) = e^{j\frac{2\pi}{\lambda}\mathbf{p}_m\cdot\mathbf{s}(\phi_k,\theta_k)}, \tag{2}$$

where λ means the wavelength, $\mathbf{p}_m \left(\in \mathbb{R}^3\right)$ is the position vector of m-th element, and $\mathbf{s}(\phi_k, \theta_k)\left(\in \mathbb{R}^3\right)$ denotes the unit directional vector of k-th signal source as

$$\mathbf{s}(\phi_k, \theta_k) = -\left[\cos\phi_k \cos\theta_k, \sin\phi_k \cos\theta_k, \sin\theta_k\right]^{\mathrm{T}}. \tag{3}$$

In addition, $\mathbf{X}\left(\in \mathbb{R}^{K\times T}\right)$ means the transmitted signal matrix over T time samples. In this paper, assuming that both Doppler frequency and carrier frequency have been pre-compensated, an element of $\mathbf{X}$ simply represents a signal power. Lastly, $\mathbf{N}\left(\in \mathbb{C}^{M\times T}\right)$ denotes the additive noise matrix for T time samples where each element follows independent Gaussian distribution with zero-mean and variance of σ^2.

III. Hierarchical MUSIC

In this section, we propose a novel hierarchical MUSIC technique for precisely estimating DOAs of multiple adjacent

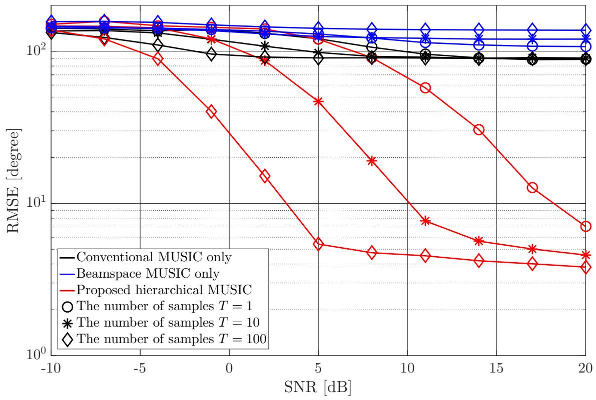

Fig. 2. RMSE performance according to SNR when two sources are from the azimuth and elevation angle pairs $(60°, 0°)$ and $(65°, 0°)$, respectively.

signal sources. Firstly, the receiver performs a coarse DOA estimation by exploiting the conventional MUSIC with received signals collected over T samples. In the conventional MUSIC, the noise subspace is separated from the covariance of the received signal to generate a spatial spectral function $P_{\mathsf{MUSIC}}(\phi, \theta)$ at an arbitrary angle set $(\phi, \theta) \in \Omega$ as follows

$$P_{\mathsf{MUSIC}}(\phi, \theta) = \frac{\mathbf{a}^{\mathrm{H}}(\phi, \theta)\,\mathbf{a}(\phi, \theta)}{\mathbf{a}^{\mathrm{H}}(\phi, \theta)\,\mathbf{E}_N\mathbf{E}_N^{\mathrm{H}}\mathbf{a}(\phi, \theta)} \tag{4}$$

where Ω represents the universal set containing all the values of the entire angle set (ϕ, θ) and $\mathbf{E}_N (\in \mathbb{C}^{M\times M-K})$ denotes the noise subspace matrix obtained by eigenvalue decomposition of the covariance. After that, by varying the angle set according to the given angular resolution, the area where the peak occurs is estimated as the area where the signal exists. Then, for the approximate area where the signal source exists, the proposed hierarchical MUSIC proceeds to the second step.

Let the area where the signal source approximately exists be $\mathcal{S}(\subset \Omega)$ and the angle set in that area $\mathcal{S}$ be $(\tilde{\phi}, \tilde{\theta})$. Then, the receiver can form $B(\leq M)$ beams within $\mathcal{S}$ and generate the beamforming matrix $\mathbf{W}(\in \mathbb{C}^{M\times B})$ as follows

$$\mathbf{W} = \left[\mathbf{a}(\tilde{\phi}_1, \tilde{\theta}_1)\ \mathbf{a}(\tilde{\phi}_2, \tilde{\theta}_2)\ \cdots\ \mathbf{a}(\tilde{\phi}_B, \tilde{\theta}_B)\right]. \tag{5}$$

In (5), each column serves to span the angular component into the corresponding beamspace. In other words, it is difficult to finely divide the entire area Ω into B beams due to the limitation of the number of beams, but in the approximate area, precise DOA estimation can be performed by spanning the signal components into the beamspace formed by $\mathbf{W}$. Therefore, in the subsequent step, the DOA can be precisely re-estimated by forming a beamspace-based spatial spectral function as

$$P_{\mathsf{BS}}(\tilde{\phi}, \tilde{\theta}) = \frac{\mathbf{a}^{\mathrm{H}}(\tilde{\phi}, \tilde{\theta})\mathbf{W}\mathbf{W}^{\mathrm{H}}\mathbf{a}(\tilde{\phi}, \tilde{\theta})}{\mathbf{a}^{\mathrm{H}}(\tilde{\phi}, \tilde{\theta})\mathbf{W}\mathbf{E}_N\mathbf{E}_N^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{a}(\tilde{\phi}, \tilde{\theta})}. \tag{6}$$

IV. Simulation Results

In this section, we considered a UCA with 10-elements in which antenna radius was half a wavelength, i.e., $d = \lambda/2$ as the receive array antenna. And, two signal sources were considered to be incident on the receiver at the azimuth and elevation angle pairs $(60°, 0°)$ and $(65°, 0°)$, respectively. Also, we set the carrier frequency to 12.7 GHz. In this simulation, two steps of the hierarchical MUSIC were considered, both steps assumed an angular resolution of 1 degree. As a performance metric, the root mean squared error (RMSE) was considered to evaluate the direction finding performance as

$$\mathrm{RMSE} = \sqrt{\frac{1}{NK}\sum_{n=1}^{N}\sum_{k=1}^{K}\left[\left(\hat{\phi}^*_{k,n} - \phi_{k,n}\right)^2 + \left(\hat{\theta}^*_{k,n} - \theta_{k,n}\right)^2\right]}, \tag{7}$$

where $\hat{\phi}^*_{k,n}, \hat{\theta}^*_{k,n}$ and $\phi_{k,n}, \theta_{k,n}$ are the estimated and true azimuth and elevation angle for the k-th source in the $n \in \{1, \cdots, N\}$ iteration, respectively.

Fig. 2 shows the RMSE performance of the proposed technique according to signal-to-noise ratio (SNR) for various numbers of samples. In the conventional MUSIC, it does not work due to the large correlation between adjacent signal sources regardless of the number of samples. Similarly, in the beamspace MUSIC, beamforming over a large search area does not help distinguish adjacent signal sources. On the other hand, our proposed hierarchical MUSIC technique shows better RMSE performance compared to the conventional MUSIC and beamspace MUSIC only even in the low signal-to-noise (SNR) regions and with small number of samples.

V. Conclusion

In this paper, we proposed a novel hierarchical MUSIC (H-MUSIC) technique with multiple receive antennas for robust direction-of-arrival (DOA) estimation even when multiple signals arrive from a similar direction. The proposed H-MUSIC consists of a coarse DOA estimation step based on the conventional MUSCI technique and a fine DOA estimation step based on the beam-space MUSIC technique. Through computer simulations, we showed that the proposed H-MUSIC elaborately estimates DOAs of multiple source signals even in the low signal-to-noise (SNR) regions and with small number of samples. In particular, the H-MUSIC can operates in a flexible structure with granular steps and variable angular resolutions.

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